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Some Results on Prime Cordial Labeling of Graphs

S. MEENA[1] & A. ARCHUNAN[2]

[1] Associate Professor of Mathematics, Govt. Arts College, C. Mutlur, Chidambaram 608 102.
[2] Associate Professor of Mathematics, Govt. Arts College, C. Mutlur, Chidambaram 608 102.

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ABSTRACT

A Prime Cordial labeling of a graph G with the vertex set $V(G)$ is a bijection f from $V(G)$ to $\{1,2,3 \dots |V(G)|\}$ such that each edge uv is assigned the label $\mathbf{1}$, if $gcd\big(f(u), f(v)\big) = 1$and 0 if $gcd\big(f(u), f(v)\big) \geq 1$ then the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1. A graph which admits a prime cordial labeling is called a prime cordial graph. In this paper, we prove that the ternary X-tree and ,$C_n^{(3)}$ are prime cordial.

INTRODUCTION

We consider a finite, connected undirected graph G = (V(G),E(G)). For standard terminology and notations we follows Bondy J.A. and Murthy USR[4]. In this section we provide brief summary of definition and the required information for our investigation.

The graph labeling is an assignment of numbers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (edges) then the labeling is called a vertex labeling (edge labeling).

Many types of labeling schemes have been introduced so far and explored as well by many researchers. Graph labelings have enormous applications within mathematics as well as to several areas of computer science and communication networks. According to Beineke and Hegde[1] graph labeling serves as a frontier between number theory and structure of graphs. For a dynamic survey on various graph labeling problems along with an extensive bibliography we refer to Gallian[5].

The study of prime numbers is of great importance as prime numbers are scattered and there are arbitrarily large gaps in the sequence of prime numbers. The notation of prime labeling was originated by Entringer and was introduced by Tout et al.[8].

After this many researchers have explored the notion of prime labeling for various graphs. Vaidya and Prajapati[10,11] have investigated many results on prime labeling. Same authors[12] have discussed prime labeling in the context of duplications of graph elements.

Motivated through the concepts of prime labeling and cordial labeling a new concepts termed as prime cordial labeling was introduced by Sundaram et al.[7] which contains blend of both the labeling.

MAIN RESULTS

Definition 2.1: (X-tree) X-tree is the graph obtained by taking five paths P_1, P_2, P_3, P_4 and P_5 of some length and identifying the end vertices of P_1, P_2, P_3 and then identifying the other end vertex of P_3 with the end vertices of P_4 and P_5.

Theorem 2.2 : Ternary X-tree is prime cordial.

Proof: Let,

$$\begin{aligned} V(G) = \{&u_1, u_2, \dots\dots u_{n-1}, u_{n+2}, \dots\dots u_{2n-1}, u_{2n+2}, \dots\dots\dots u_{3n-1},\\ &v_1, v_2, \dots\dots v_{n-1}, v_{n+2}, \dots\dots v_{2n-1}, v_{2n+2}, \dots\dots v_{3n-1}, w_1\\ &= v_n = w_1, w_2, \dots. w_n\\ &= u_{2n}, v_{2n}, w_{n+1}, w_{n+2}, \dots\dots w_{2n-1}, w_{2n} = u_{3n}\\ &= v_{n+1}, w_{2n+1}, \dots. w_{3n-1}, w_{3n} = V_{2n} = u_{n+1}\} \end{aligned}$$

and

$E(G) = \{u_i u_{i+1} / 1 \le i \le n-1,\ n+1 \le i \le 2n \text{ and } 2n+1 \le i \le 3n\} \cup \{v_i v_{i+1} / 1 \le i \le n-1,\ n+1 \le i \le 2n \text{ and } 2n+1 \le i \le 3n\} \cup \{w_i w_{i+1} / 1 \le i \le n-1,\ n+1 \le i \le 2n \text{ and } 2n+1 \le i \le 3n\}$

Case (i): n is even

Define a labeling $f: V(G) \to \{1, 2, 3, \dots\dots..9(n-1)+1\}$

by

$$\begin{aligned} f(u_i) &= 2i-1 \text{ for } 1 \le i \le n-1\\ f(v_i) &= 2i \quad \text{for } 1 \le i \le n-1\\ f(w_i) &= 2n + (2i-2) \text{ for } 1 \le i \le \left\lfloor \tfrac{n}{2} \right\rfloor\\ f(w_n) &= 3n-3,\\ f(w_{n-i}) &= f(w_n) - 2i \quad \text{for } 1 \le i \le \left\lfloor \tfrac{n}{2} \right\rfloor - 1\\ f(v_{n+1}) &= 3n,\\ f(v_i) &= f(v_{i-1}) + 2 \quad \text{for } n+2 \le i \le 2n-1\\ f(u_{n+1}) &= 3n-1,\\ f(u_i) &= f(u_{i-1}) + 2 \quad \text{for } n+2 \le i \le 2n-1\\ f(w_{n+1}) &= 5n-2,\\ f(w_i) &= f(w_{i-1}) + 2 \quad \text{for } n+2 \le i \le 2n-1\\ f(v_{2n+1}) &= 5n-3,\\ f(u_i) &= f(u_{i-1}) + 2 \quad \text{for } 2n+2 \le i \le 3n-1\\ f(v_{2n+1}) &= 7n-5,\\ f(v_i) &= f(v_{i-1}) + 2 \quad \text{for } 2n+2 \le i \le 3n-1\\ f(w_{2n+1}) &= 7n-4,\\ f(w_i) &= f(w_{i-1}) + 2 \quad \text{for } 2n+2 \le i \le 3n-1 \end{aligned}$$

Case (ii): if n is odd $n \ge 9$, $n+1 \not\equiv 0 (mod\ 3)$ and $n+1 \equiv 0 (mod\ 10)$

Define a labeling $f: V(G) \to \{1, 2, 3 \dots\dots\dots 9(n-1)+1\}$ by

$$f(u_i) = 2i-1 \qquad \text{for } 1 \le i \le n-1$$

$f(v_i) = 2i$ for $1 \le i \le n-1$

$f(w_i) = 2n + (2i-2)$ for $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$

$f(w_n) = 3n - 2$

$f(w_{n-i}) = f(w_n) - 2i$ for $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$

$f(v_{n+1}) = 3n - 1$

$f(v_i) = f(v_{i-1}) + 2$ for $n+2 \le i \le 2n-1$

$f(u_{n+1}) = 3n$

$f(u_i) = f(u_{i-1}) + 2$ for $n+2 \le i \le 2n-1$

$f(w_{n+1}) = 5n - 3$

$f(w_i) = f(w_{i-1}) + 2$ for $n+2 \le i \le 2n-1$

$f(u_{2n+1}) = 5n - 2$

$f(u_i) = f(u_{i-1}) + 2$ for $2n+2 \le i \le 3n-1$

$f(v_{2n+1}) = 7n - 4$

$f(v_i) = f(v_{i-1}) + 2$ for $2n+2 \le i \le 3n-1$

$f(w_{2n+1}) = 7n - 5$

$f(w_i) = f(w_{i-1}) + 2$ for $2n+2 \le i \le 3n-1$

Case (iii) If n is odd $n \ge 9$, $n+1 \equiv 0(mod3)$ and $n+1 \not\equiv 0(mod10)$

Interchange the labels of w_{n-2} and $W_{\left\lfloor \frac{n}{2} \right\rfloor + 2}$, in case (ii)

Case (iv): If n is odd and $n \equiv 0(mod3)$

Define a labeling $f: V(G) \rightarrow \{1, 2, 3, \ldots..9(n-1)+1\}$

$f(u_i) = 2i - 1$ for $1 \le i \le n-1$

$f(v_i) = 2i$ for $1 \le i \le n-1$

$f(w_i) = 2n + (2i-2)$ for $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$

$f(w_n) = 3n - 2$

$f(w_{n-i}) = f(w_n) - 2i$ for $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$

$f(v_{n+1}) = 3n - 1$

$f(v_i) = f(v_{i-1})$ for $n+2 \le i \le 2n-1$

$f(u_{n+1}) = 3n$

$f(u_i) = f(u_{i-1}) + 2$ for $n+2 \le i \le 2n-1$

$f(w_{n+1}) = 5n - 3$

$f(w_i) = f(w_{i-1}) + 2$ for $n+2 \le i \le 2n-1$

$f(u_{2n+1}) = 5n - 2$

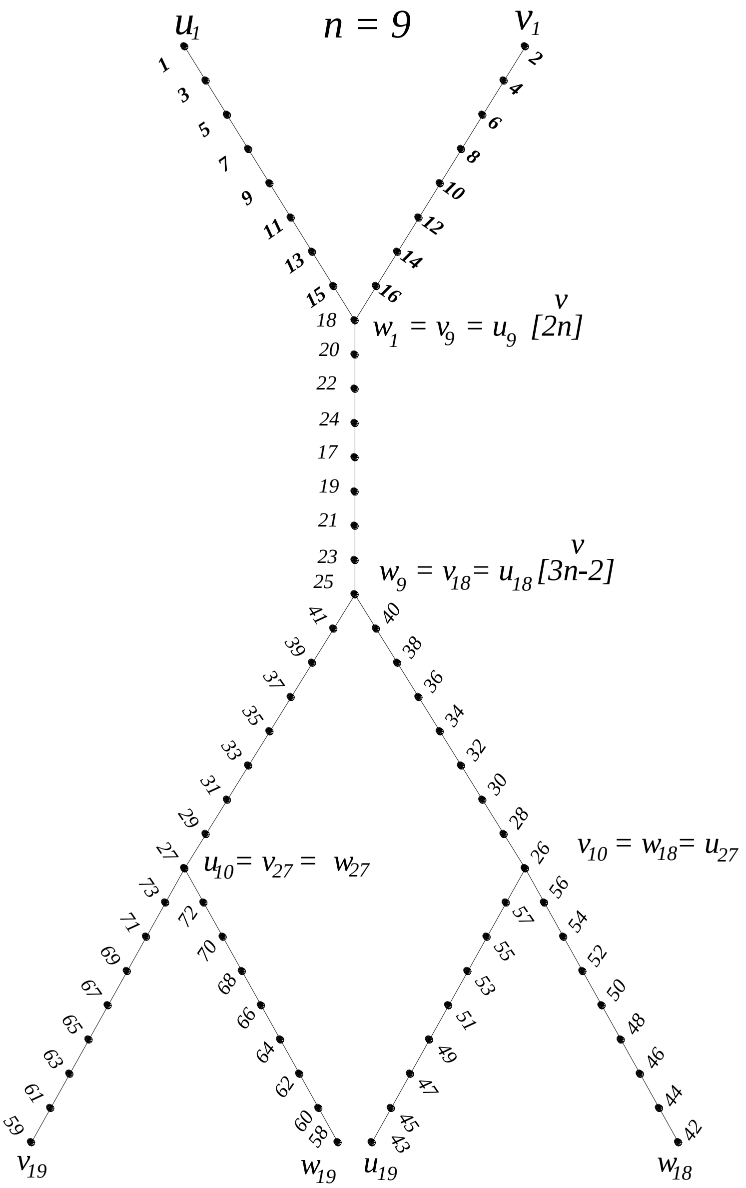
$f(v_i) = f(u_{i-1}) + 2$ for $2n+2 \le i \le 3n-1$

$f(v_{2n+1}) = 7n - 4$

$f(v_i) = f(v_{i-1}) + 2$ for $2n+2 \le i \le 3n-1$

$f(w_{2n+1}) = 7n - 5$

$f(w_i) = f(w_{i-1}) + 2$ for $2n+2 \le i \le 3n-1$



Theorem: 2.3 The graph $C_n^{(3)}$ is prime cordial labeling, where n is odd and $n \geq 5$.

Proof: Let $V(G) = \{u_1, u_2, \dots \dots u_{n-1},\ v_1, v_2, \dots \dots v_{n-1}, w_1, w_2, \dots \dots w_{n-1}, u_n = v_n = w_n\}$

$$E(G) = \{u_i u_{i+1}, v_i v_{i+1}, w_i w_{i+1} / 1 \leq i \leq n-1\}$$

Define a labeling $f: V(G) \to \{1, 2, 3, \dots .3n-2\}$

Let $u_n = v_n = w_n = 3n - 2$

$$f\left(u_{\frac{n}{2}}\right) = 1$$

$$f(u_i) = n - 1 - 2i \quad \text{for } 1 \leq i \leq \left[\tfrac{n}{2}\right] - 1$$

$$f(u_i) = 4n - 2i - 1 \quad \text{for } \left[\tfrac{n}{2}\right] + 1 \leq i \leq n - 1$$

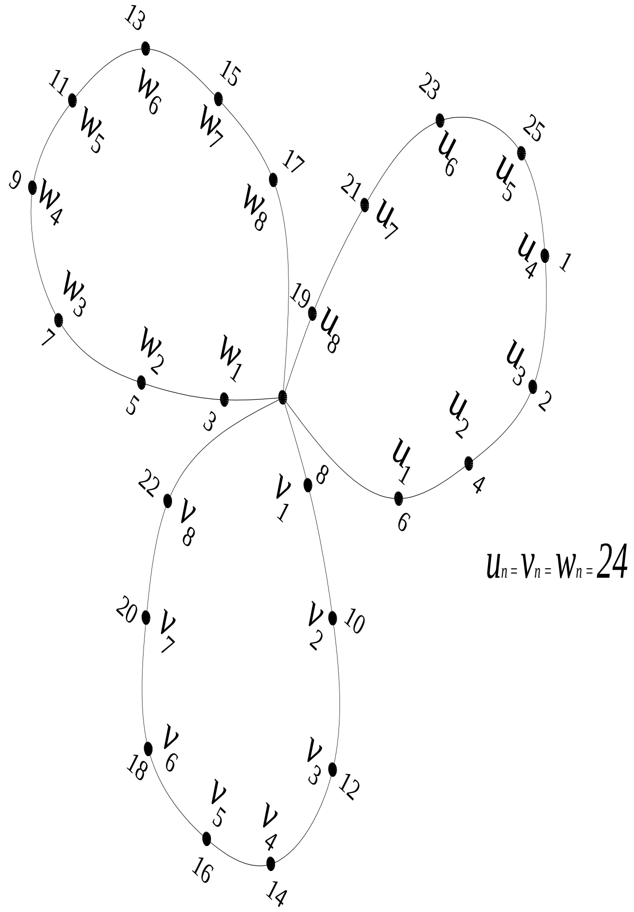
$$f(v_i) = n + 2i - 3 \quad \text{for } 1 \leq i \leq n - 1$$

$$f(w_i) = 2i + 1 \quad \text{for } 1 \leq i \leq n - 1$$

This satisfies $|e_f(0) - e_f(1)| \leq 1$.

Hence $C_n^{(3)}$ is prime cordial.

$C_9^{(3)}$



Theorem: 2.4 Cycle C_n, with five chords (makes double triangle),n ≡ 0(mod 3) is prime cordial (n is even) n ≥ 12.

Proof: Case 1 Let G by the Cycle C_n, n is even, n ≥ 12.

Let u_1,u_2 ...... u_n be the vertices of cycle C_n.

Let $e'_1 = u_1 u_3$, $e'_2 = u_3 u_{n-1}$, $e'_3 = u_3 u_{\lfloor \frac{n}{2} \rfloor +1}$, $e_4^1 = u_3 u_{\lfloor \frac{n}{2} \rfloor +1}$ $u_{n-1} e_5^1 = u_1 u_{n-1}$ be chords of the cycle C_n.

We define the labeling f:V(G) →{1,2, ...n} as follows :

$f(u_1) = 2$, $f(u_2) = 4$, $f(u_3) = 6$, $f(u_n)=1$

Label the consecutive vertices u_i, $4 \le i \le \frac{n}{2}$ by 8, 10, ...n. and label the remaining vertices u_i, where $\frac{n}{2} + 1 \le i \le n - 1$. by 3,5,7,9........n-1

In view of the above labeling pattern, we have

$$|e_f(0) - e_f(1)| \le 1.$$

Hence C_n with five chord is prime cordial.

Case 2: The Cycle $C_n$with five chords (makes double triangle) is prime cordial if n is even (except n ≅ 0 (mod 3))

Here inter change $u_{n-1}$by u_{n-3}

in view of the above labeling pattern, we have

$$|e_f(0) - e_f(1)| \le 1.$$

Hence C_n with five chord is prime cordial

Theorem : 2.5 The Cycle $C_n$with five chords (makes double triangle) is prime cordial if n is odd, n ≅0 (mod 3) n ≥ 9 and n ≅ 2(mod 3)

Proof: Let G by the Cycle c_u, n is even n ≥ 9.

Let u_1, u_2 ...... u_n, be the vertices of cycle c_n.

Let $e'_1 = u_1 u_3$, $e'_2 = u_3 u_{n-1}$

$e'_3 = u_{\lfloor \frac{n}{2} \rfloor + 1} u_{n-1}$, $e_4^1 = u_3 u_{\lfloor \frac{n}{2} \rfloor + 1}$ and $e_5^1 = u_1 u_{n-1}$ be the chords of the cycle $c_n$we define the labeling by f: V(G) → {1,2, ...n}as follows

$f(u_1) = 2$, $f(u_2) = 4$, $f(u_3) = 6$, $f(u_n)=1$.Label the consecutive veritiesui, $4 \le i \le \lfloor \frac{n}{2} \rfloor$ and label the remarking vertices ui, $\lfloor \frac{n}{2} \rfloor + 1 \le i \le n$. by 3,5,7,9........n

in view of the above labeling pattern, we have

$$|e_f(0) - e_f(1)| \le 1.$$

Hence C_n with five chord is prime cordial

CONCLUSION

By using a property from number theory, we examined the existence of prime cordial labeling of graphs. To investigate analogous results for different graphs as well as in the context of various graph labeling problems is an open area of research.

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